Maa Omwati Degree's College Hasanpur

Exam Notes

Course: M.com 3rd sem.

Subject: Fundamental Of Economics

UNIT-I

Concept of Stochastic Processes

A **stochastic process** is a mathematical concept used to describe a collection of random variables that evolve over time (or space). In simple terms, it is a process that involves randomness and uncertainty in its outcomes.

Key Points:

1. **Definition**:

2. A stochastic process is a family of random variables {Xt:t∈T}\{X_t : t \in T\}, where:

    a. tt represents the index (often time or space),
    b. TT is the index set (e.g., discrete time: t=0,1,2,…t=0,1,2,\dots or continuous time: t≥0t \geq 0),
    c. XtX_t represents the value of the process at time tt, which is random.
3. **Deterministic vs Stochastic**:
    a. A **deterministic process** has outcomes that are predictable and fixed.
    b. A **stochastic process** has outcomes that involve randomness and can vary in repeated experiments.
4. **Examples**:
    a. **Random Walk**: Movement where each step is randomly determined.
    b. **Markov Chain**: A stochastic process where the next state depends only on the current state, not the past history.
    c. **Poisson Process**: Models random events occurring over time (e.g., arrival of phone calls).
    d. **Brownian Motion**: Continuous-time stochastic process used in physics and finance.
5. **Applications**:
    a. Economics and Finance (stock price modeling, risk analysis)
    b. Physics (particle motion, thermodynamics)
    c. Engineering (signal processing, reliability studies)
    d. Biology (population dynamics, genetics)

- **In short:**

A stochastic process is a **mathematical model for systems or phenomena that evolve over time in an uncertain or random way**.

Concept of Unit Root Stochastic Processes

A **stochastic process** is a sequence of random variables evolving over time. When studying time series in economics, finance, or statistics, one important property is **stationarity**—whether the mean, variance, and covariance remain constant over time.

A **unit root process** is a special type of stochastic process where:

1. The process is **non-stationary** (its statistical properties change over time).
2. It has a **unit root** in its characteristic equation (i.e., the autoregressive coefficient equals 1).

Mathematical Form

Consider a simple **autoregressive process of order 1 (AR(1)):**

$$y_t = \rho y_{t-1} + \epsilon_t$$

- If $|\rho| < 1$, the process is **stationary**.
- If $\rho = 1$, the process becomes:

$$y_t = y_{t-1} + \epsilon_t$$

This is called a **random walk**, which is the simplest form of a unit root process.

Here, ϵ_t is a white noise error term.

Key Features of Unit Root Processes

- **Non-stationary**: Mean and variance are not constant.
- **Shocks have permanent effects**: A disturbance in the series affects all future values (unlike stationary processes, where shocks die out).
- **Difficult to forecast long-term**: Because the process "wanders" without reverting to a mean.

- Often appear in **economic and financial time series** (GDP, stock prices, exchange rates, etc.).

Importance

- Detecting unit roots (using tests like the **Dickey-Fuller** or **Augmented Dickey-Fuller test**) is crucial in econometrics.
- Many statistical methods require stationarity, so non-stationary unit root processes must be transformed (usually by differencing) before analysis.

 In short:

A **unit root stochastic process** is a non-stationary time series where shocks have a lasting impact, typically modeled as a random walk. It's central in econometrics because ignoring it can lead to misleading inferences.

Concept of Non-Stationary Time Series

A **non-stationary time series** is a sequence of data points measured over time whose **statistical properties such as mean, variance, and autocorrelation change over time**.

- In contrast to a **stationary time series**, where these properties remain constant, non-stationary series often show **trends, seasonality, cycles, or structural breaks**.
- Because of these changing characteristics, non-stationary series are more difficult to model and forecast accurately without first transforming them into stationary form (e.g., by differencing, detrending, or applying logarithms).

Key Features of Non-Stationary Time Series

1. **Changing Mean** – The average value shifts over time (e.g., upward trend in GDP).
2. **Changing Variance** – Fluctuations grow or shrink over time (e.g., volatility in stock prices).
3. **Changing Autocorrelation** – The dependence of current values on past values is not constant.
4. **Presence of Trend/Seasonality** – Long-term direction or repetitive patterns exist.

Examples

- GDP of a country (shows growth trend).
- Stock market index (variance and volatility change over time).
- Temperature data (shows seasonality).

– In practice, econometricians and statisticians often convert a non-stationary time series into a **stationary series** before analysis, using methods like **differencing** (ARIMA models), **detrending**, or **log transformation**.

Here's a **clear definition of the concept of Unit Root Test**:

A **Unit Root Test** is a statistical test used in time series analysis to check whether a time series is **stationary** or **non-stationary**.

- A time series is **stationary** if its mean, variance, and autocovariance remain constant over time.
- If a time series has a **unit root**, it means the series is **non-stationary** and exhibits a random walk, making forecasting and modeling more difficult.

The unit root test helps determine whether **differencing** (or other transformations) is needed to make the data stationary before applying econometric models like ARIMA or regression analysis.

Common Unit Root Tests:

1. **Augmented Dickey-Fuller (ADF) Test**
2. **Phillips-Perron (PP) Test**
3. **KPSS Test** (tests stationarity instead of non-stationarity)

In short:

A unit root test tells us if shocks to a time series have **temporary effects** (stationary) or **permanent effects** (non-stationary).

Concept of Structural Breaks in Time Series

A **structural break** in a time series occurs when there is an **unexpected shift in the underlying data-generating process** at a certain point in time. This shift alters the statistical properties of the series — such as mean, variance, or trend — leading to a change in the overall pattern of the data.

In simple terms, it means the behavior of the series before the break is significantly different from the behavior after the break.

Key Features of Structural Breaks

1. **Sudden change** – It may occur due to policy changes, economic crises, technological innovations, wars, natural disasters, etc.
2. **Parameter shift** – Coefficients of a model (e.g., slope, intercept in regression) change after the break.
3. **Types of breaks**:
    a. **Level shift**: Sudden jump/drop in the mean level of the series.
    b. **Trend break**: Change in the growth path or slope of the series.
    c. **Variance break**: Change in volatility of the series.

Examples

- An economy's GDP series showing a sudden decline during a financial crisis.
- Stock prices showing higher volatility after a major regulatory change.
- Inflation series changing trend after adoption of a new monetary policy.

Importance

- Ignoring structural breaks can lead to **biased estimates and poor forecasts**.
- Properly detecting and modeling breaks improves the reliability of time series analysis.

UNIT-II

Here's a clear explanation of the concept of **Autoregressive Distributed Lag (ARDL) models**:

An **Autoregressive Distributed Lag (ARDL) model** is a type of econometric model that combines both **autoregressive terms (lags of the dependent variable)** and **distributed lag terms (lags of the independent variables)**.

It is widely used to analyze the **dynamic relationship** between variables, especially when the data series are of **different orders of integration** (some stationary at level I(0), some at first difference I(1), but not I(2)).

The ARDL model is particularly useful for:

1. **Short-run dynamics**: It shows how variables adjust in the short term.
2. **Long-run relationships (cointegration)**: Through the ARDL bounds testing approach, it helps check whether variables are cointegrated (i.e., move together in the long run).

General form of ARDL(p, q):

$$Y_t = \alpha_0 + \sum_{i=1}^{p} \beta_i Y_{t-i} + \sum_{j=0}^{q} \delta_j X_{t-j} + \mu_t$$

Where:

- Y_t = dependent variable
- X_{t} = independent variable
- p, q = number of lags
- β_i = coefficients of lagged dependent variable (autoregressive part)
- δ_j = coefficients of lagged independent variable (distributed lag part)
- μ_t = error term

In short:

ARDL models are powerful tools in econometrics to capture both short-run and long-run relationships among variables, even when data series are a mix of stationary and non-stationary types.

Concept of Co-integration

Co-integration is a statistical concept used in time series analysis. It refers to the situation where two or more non-stationary time series (which individually may wander over time) are linked by a long-run equilibrium relationship.

- Normally, a non-stationary series has a mean and variance that change over time, making it difficult to predict.

- However, if a certain linear combination of these non-stationary series becomes **stationary** (i.e., it has a constant mean and variance over time), then the series are said to be **co-integrated**.
- This means that although the individual series may drift apart in the short run, they move together in the long run, maintaining a stable relationship.

Example:

Suppose stock prices of two companies often move randomly. Individually, they are non-stationary. But if their prices have a long-run relationship (for example, the price difference remains relatively stable), then they are co-integrated.

Let's define **causality in time series** in a clear way:

Causality in Time Series

In time series analysis, **causality** refers to a statistical relationship where changes in one variable help to predict changes in another variable over time. The most common way to measure this is through **Granger causality**.

- **Granger Causality Concept**:
- A variable **X** is said to "Granger-cause" another variable **Y** if past values of **X** contain information that helps to predict the future values of **Y**, beyond the information contained in past values of **Y** alone.
    - It does **not** mean true cause-and-effect in a philosophical sense, but rather **predictive causality**.

Example:

If past interest rates help to predict future inflation (even after considering past inflation), then interest rates Granger-cause inflation.

Here's a clear explanation for you:

Impulse Response Analysis – Concept

Impulse Response Analysis is a method used in economics, finance, and signal processing to study how a system or a variable reacts over time to an external shock (or impulse). In simple terms, it shows the path of adjustment of a variable when there is a sudden, one-time disturbance in the system.

- **Purpose:** To understand the dynamic relationship between variables.
- **Use in Economics/Finance:** In Vector Auto Regression (VAR) models, it helps measure how one economic variable (like inflation, GDP, or interest rates) responds to a shock in another variable.
- **Key Point:** It does not just measure the immediate impact but also the future effects (short-run and long-run responses).

Example:

If there is an unexpected increase in interest rates (shock), impulse response analysis shows how GDP, investment, or inflation will move immediately and in the following periods.

UNIT-III

Here's a clear explanation of the **Introduction of ARMA Models**:

The **ARMA model** stands for **Autoregressive Moving Average model**. It is a statistical model used for analyzing and forecasting time series data.

- **Autoregressive (AR) part**: This part states that the current value of the time series depends linearly on its past values.
- **Moving Average (MA) part**: This part expresses that the current value of the series depends on past error terms (random shocks).
- **ARMA model**: When both AR and MA components are combined, the model is called ARMA.

It is most suitable for **stationary time series data**, where the mean and variance remain constant over time. ARMA models are widely used in economics, finance, weather forecasting, and many other fields for predicting future values based on past observations.

Here's a clear explanation of the **concept of properties of ARMA models**:

ARMA Model (Autoregressive Moving Average Model):

An ARMA model is a statistical model used to analyze and forecast time series data. It combines two components:

- **AR (Autoregressive part):** The current value of the series depends on its own past values.
- **MA (Moving Average part):** The current value also depends on past error terms (shocks or disturbances).

Key Properties of ARMA Models

1. **Stationarity:**
    a. ARMA models assume that the time series is *weakly stationary*, meaning its mean, variance, and autocovariance do not change over time.
    b. This ensures that the series fluctuates around a constant mean.
2. **Linearity:**
    a. ARMA models are *linear models*, where the current value of the series is expressed as a linear combination of past observations and past error terms.
3. **Dependence Structure:**
    a. AR part explains the *long-term dependence* in the series.
    b. MA part explains the *short-term shocks* or randomness.
4. **Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF):**
    a. In AR models, PACF cuts off after a lag equal to the order of AR.
    b. In MA models, ACF cuts off after a lag equal to the order of MA.
    c. In ARMA models, both ACF and PACF decay gradually.
5. **Invertibility:**
    a. For the MA part, the model must be invertible (ensuring a unique representation in terms of AR components).
6. **Parsimonious Representation:**
    a. ARMA models capture complex time series behavior with fewer parameters compared to using only AR or only MA.
7. **Forecasting Ability:**
    a. ARMA models are widely used in forecasting because they can represent both persistence (trends from the AR part) and shocks (from the MA part).

b.

Here's a clear explanation of the **concept of estimation of ARMA models**:

Estimation of ARMA Models refers to the process of finding the most suitable values of the parameters in an **Autoregressive Moving Average (ARMA)** model so that it can represent the underlying time series data accurately.

- An **ARMA(p, q) model** combines two components:
  - **AR (Autoregressive part):** Relates the current value of the series to its past values.
  - **MA (Moving Average part):** Relates the current value to past forecast errors (shocks).
- The **estimation process** involves:
  - **Model Identification:** Selecting the appropriate order (p, q) using tools like **Autocorrelation Function (ACF)** and **Partial Autocorrelation Function (PACF)**.
  - **Parameter Estimation:** Estimating AR and MA coefficients, usually through methods such as:
    - **Maximum Likelihood Estimation (MLE)**
    - **Conditional Least Squares**
  - **Diagnostic Checking:** Verifying whether the residuals (errors) behave like white noise (no autocorrelation left).

The goal is to obtain parameter estimates that minimize forecast errors and best capture the data's dynamics.

Here's a clear explanation of the **Introduction of ARIMA Models**:

ARIMA (AutoRegressive Integrated Moving Average) model is a statistical method used for analyzing and forecasting time series data. It combines three important components:

1. **AutoRegressive (AR):** Refers to the relationship between an observation and its previous values.
2. **Integrated (I):** Refers to the process of making a time series stationary (removing trend and seasonality) by differencing.
3. **Moving Average (MA):** Refers to the relationship between an observation and the residual errors from a moving average model applied to lagged observations.

An ARIMA model is generally denoted as **ARIMA(p, d, q)**, where:

- **p** = number of lag observations (AR order),
- **d** = degree of differencing (to make the series stationary),
- **q** = size of the moving average window (MA order).

Purpose: ARIMA is widely used in economics, finance, business, weather prediction, and many fields where future forecasting is required based on past trends. It is powerful because it captures different aspects of time series patterns (trend, autocorrelation, and randomness).

Here's a clear explanation of the **concept and properties of ARIMA models**:

Concept of ARIMA Models

ARIMA (AutoRegressive Integrated Moving Average) is a widely used statistical model for analyzing and forecasting time series data.

- **AR (AutoRegressive part):** Indicates that the variable of interest is regressed on its own past values.
- **I (Integrated part):** Refers to differencing the data to make it stationary (removing trends and seasonality).
- **MA (Moving Average part):** Represents the dependency between a time series observation and a residual error from a moving average model applied to lagged observations.

The ARIMA model is generally denoted as **ARIMA(p, d, q)** where:

- **p** = order of autoregression
- **d** = degree of differencing (to achieve stationarity)
- **q** = order of moving average

Properties of ARIMA Models

1. **Stationarity**
    a. ARIMA models assume that the time series is stationary (constant mean, variance, and autocovariance over time).

   b. Non-stationary data is made stationary using differencing (the "I" part).
2. **Linearity**
   a. ARIMA is a linear model, meaning the current value of the series is expressed as a linear function of past values and past errors.
3. **Dependence on Past Values s Errors**
   a. The AR part captures correlation with past values, while the MA part captures correlation with past forecast errors.
4. **White Noise Residuals**
   a. After fitting a correct ARIMA model, the residuals (errors) should resemble white noise (random, no autocorrelation).
5. **Flexibility**
   a. ARIMA can model a wide variety of time series patterns, including trends and short-term dependencies.
6. **Forecasting Ability**
   a. ARIMA is particularly effective for short-term forecasting due to its reliance on past data patterns.
7. **Parsimony**
   a. The goal is to use the smallest possible values of (p, d, q) to adequately model the time series without overfitting.
8. **Parameter Estimation**
   a. Parameters are usually estimated using methods like Maximum Likelihood Estimation (MLE) or Least Squares.

G. **Extension Possibilities**
   a. ARIMA can be extended to:
      i. **SARIMA (Seasonal ARIMA)** for seasonal data.
      ii. **ARIMAX** when explanatory variables are included.

□ In short: **An ARIMA model explains a time series based on its past values (AR), past errors (MA), and differencing (I), with the key properties being stationarity, linearity, white noise residuals, and strong forecasting capability.**

Here's a **clear explanation of the concept of Estimation of ARIMA Models**:

Estimation of ARIMA Models refers to the process of identifying and calculating the parameters of an ARIMA (AutoRegressive Integrated Moving Average) model that best fit a given time series data.

An ARIMA model is generally represented as **ARIMA(p, d, q)**, where:

- **p** = order of the autoregressive (AR) part,
- **d** = degree of differencing to make the series stationary,
- **q** = order of the moving average (MA) part.

The **estimation process** involves three key stages:

1. **Identification:**
    a. Determine whether the series is stationary (if not, apply differencing).
    b. Use tools like the Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) to tentatively identify the values of *p* and *q*.
2. **Estimation of Parameters:**
    a. Once *p, d, q* are chosen, the parameters of the AR and MA components are estimated.
    b. Methods such as **Maximum Likelihood Estimation (MLE)** or **Least Squares Estimation** are typically used.
3. **Diagnostic Checking:**
    a. After estimation, the residuals (errors) are checked to ensure they behave like white noise (i.e., random with no pattern).
    b. Goodness-of-fit criteria like **Akaike Information Criterion (AIC)** or **Bayesian Information Criterion (BIC)** are often used to compare models and select the best one.

In short, **estimation of ARIMA models** is about finding the correct structure (p, d, q) and parameter values so that the model accurately represents and forecasts the underlying time series data.

UNIT-IV

Here's a clear explanation for you:

Introduction to Panel Data Models

Concept:

Panel data models are statistical and econometric models that deal with **data containing observations on multiple entities (like individuals, firms, states, or countries) across time**. Such data is called **panel data** (also known as longitudinal data).

- **Structure:** Panel data has two dimensions — **cross-sectional (different entities)** and **time-series (different periods)**.
- **Purpose:** Panel data models help analyze both the differences across entities and the changes within entities over time.
- **Types:**
  - **Pooled OLS Model** – treats panel data as simple pooled cross-sectional data, ignoring time and individual differences.
  - **Fixed Effects Model** – controls for unobserved individual characteristics that do not change over time.
  - **Random Effects Model** – assumes individual-specific effects are random and uncorrelated with explanatory variables.

Importance:

- Provides more **information, variability, and efficiency** compared to pure cross-sectional or time-series data.
- Helps control for **individual heterogeneity** (differences across units).
- Useful in economics, social sciences, and policy research for understanding dynamic behavior over time.

Here's a clear explanation of **Pooled OLS (Ordinary Least Squares)**:

Concept of Pooled OLS

Pooled OLS is a basic method used in panel data analysis. Panel data means data collected over time for the same individuals, firms, states, or countries. Instead of separating the cross-sectional (individual differences) and time-series (changes over time) aspects, pooled OLS simply combines ("pools") all observations together and estimates a single regression line using the ordinary least squares method.

It assumes:

1. There are no significant individual-specific or time-specific effects.
2. The error term captures all unobserved heterogeneity.

3. The relationship between dependent and independent variables is the same across individuals and time.

Because it ignores unobserved individual or time effects, pooled OLS is often **too simplistic** for panel data and may lead to **biased or inconsistent results** if those effects actually exist. That is why it is usually compared with more advanced models like **Fixed Effects** or **Random Effects**.

Here's a clear explanation of the **Fixed Effects Model**:

Fixed Effects Model - Concept

A **Fixed Effects Model (FEM)** is a statistical model commonly used in **panel data analysis** (data that combines time series and cross-sectional dimensions).

It assumes that **individual-specific characteristics (effects) that may influence the dependent variable are constant (fixed) over time** but may differ across individuals (such as countries, firms, or people). By controlling for these unobserved characteristics, the model focuses only on estimating the impact of explanatory variables that change over time.

- In other words, FEM removes the effect of **time-invariant factors** (like culture, geography, or inherent traits) so that the estimated coefficients reflect the pure effect of variables that change over time.
- This is achieved by using **within transformation (demeaning method)** or **dummy variables** for each entity.

Key Points

1. **Purpose:** To control for unobserved heterogeneity across individuals when this heterogeneity is constant over time.
2. **Assumption:** Individual-specific effects are correlated with the explanatory variables.
3. **Method:** Uses entity-specific intercepts (fixed constants) to capture unique characteristics of each individual/unit.
4. **Equation:**

$$Y_{it} = \alpha_i + \beta X_{it} + \varepsilon_{it}$$

where

a. Y_{it} = dependent variable for unit *i* at time *t*
b. α_i = individual-specific intercept (fixed effect)
c. X_{it} = explanatory variables
d. β = coefficients to be estimated
e. ε_{it} = error term

Example

Suppose we want to study the effect of **training programs on worker productivity** over time.

- Factors like **innate ability** or **education level** of workers may not change much with time but can strongly influence productivity.
- FEM controls for these fixed differences, so the analysis focuses only on changes within the same worker over time.

– – In short: **The Fixed Effects Model is used to analyze panel data by controlling for unobserved, time-invariant individual effects, allowing us to focus on the impact of variables that vary over time.**

Here's a clear explanation of the **Random Effects Model**:

Concept of Random Effects Model

The **Random Effects Model (REM)** is a statistical and econometric approach used in panel data analysis, where data involve both cross-sectional (different individuals, firms, countries, etc.) and time-series dimensions.

In this model, it is assumed that individual-specific effects (differences across entities) are **randomly distributed** and uncorrelated with the explanatory (independent) variables. Unlike the Fixed Effects Model, which treats these effects as constant parameters, the Random Effects Model treats them as random variables drawn from a larger population.

This makes REM more efficient when its assumptions hold true, as it allows both within-entity (over time) and between-entity (across individuals) variations to explain outcomes.

Key Features:

1. Assumes individual-specific effects are random, not fixed.
2. These effects are uncorrelated with the explanatory variables.
3. Suitable when sample units are randomly drawn from a larger population.
4. Estimated using techniques like Generalized Least Squares (GLS).

Example:

If we study the effect of education and experience on wages across different workers over several years, the unobserved characteristics of workers (like talent or motivation) are treated as random effects.